A ROBUST AND COMPUTATIONALLY EFFICIENT METHOD FOR TONAL ACTIVE NOISE CONTROL USING A SIMPLIFIED SECONDARY PATH MODEL

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ABSTRACT

We present a computationally efficient method for sinusoidal noise cancellation based on the FXLMS algorithm. It features subsampling in order to increase convergence speed and decrease computational requirements, and most importantly, does not require extra noise added to the filter output for secondary path identification. In addition, it is robust to secondary path variations and in low SNR scenarios, which are frequently found in practical active noise control systems, features fast tracking and can be directly generalized to multichannel systems. We illustrate its operation with simulations.

Index Terms— Active noise control, acoustic noise, sinusoidal noise cancellation, filtered-X least mean square methods, secondary path model.

1. INTRODUCTION

Tonal noise is present in many noise control scenarios. It is typically found in applications featuring periodic driving signals, such as power transformers and rotating machinery. To perform active noise control (ANC), one would like to generate a signal which interferes destructively with the original noise, thereby cancellating it. To adaptively model the noise-generating mechanism, one often resorts to the least-mean squares (LMS) algorithm [1].

In practical applications, the secondary path (SP) defined from the controller output to the cancellation points is not trivial: it incorporates a D/A converter, amplifiers, transducer gains and distortions, an acoustic delay, reflections, and an A/D converter. The FXLMS algorithm [2] compensates for the secondary path transfer function, and as long as the SP is correctly modeled and the adaptation step is sufficiently small, the FXLMS filter weights converge to the Wiener solution [3]. As shown in [4], this is also true for the complex FXLMS and multitonal noise.

The requirement to correctly model the SP often demands more time and resources than the cancellation main loop. Acoustic impulse responses can be very long, resulting in high computational requirements and slow convergence of the SP identifier. Moderate errors in the SP model lead to slow convergence for the control filter, and large errors can cause the controller to diverge. To complicate matters, most general-purpose online SP identification and tracking algorithms use a system identification configuration, and therefore require auxiliary noise to be added to the controller output, increasing the residual noise power. For open-field control of power transformer noise, the task is particularly difficult: a large number of transducers is necessary (i.e., several SP models must be estimated in parallel), and the error microphones must be placed far away from the control loudspeakers (so the SNR for a conventional SP identification filter will be very low, resulting in very slow convergence).

In order to mitigate or circumvent these issues, several alternatives have been proposed. For example, [5, 6, 7] improve the system identification configuration with auxiliary adaptive filters, auxiliary noise power control and variable step sizes. Even though these modifications increase overall robustness and convergence characteristics, they have higher computational costs. The requirement to use long filters makes wideband SP estimation an inherently complex step that one would like to avoid. This is specially true for narrowband or tonal ANC, where a model valid only for the frequency band of the noise suffices.

A few methods require no secondary path identification. [8] proposes a frequency-domain perturbation method with a variable step size to update the adaptive filter. For modest perturbations, convergence is much slower than for LMS-based methods. The method proposed in [9] uses 3 LMS filters for the complete ANC system. Since these filters are expected to be long, this solution is also computationally demanding.

It is a necessary condition for FXLMS stability in the tonal noise scenario that the estimated SP phase error remain below $\pm 90^\circ$ [2]. Based on this observation, [10] proposes monitoring the residual noise power and switching the sign of the adaptation coefficient if divergence is detected. This sign change is equivalent to a 180° phase shift, which is enough to satisfy the $\pm 90^\circ$ bound and results in convergence for sufficiently small adaptation steps. However, as the SP estimated phase error increases, the maximum allowable step size and convergence speed decrease. If the phase error is close to the bound, the tonal FXLMS will not have acceptable tracking performance. This method also has no provision for detecting phase changes before divergence is experienced.

In the following sections we introduce an FXLMS algorithm for single-tone noise which only requires estimation of the secondary path delay. It features subsampling to improve convergence speed and decrease computational requirements, online tracking to detect slow secondary path variations and can be easily generalized to a multi-channel scenario.

2. PROBLEM FORMULATION

Fig. 1 shows a block diagram for the FXLMS algorithm in a feedforward configuration. In this section, $P(z)$ is the main acoustic path, $S(z)$ is the secondary path, $\hat{S}(z)$ is the estimated secondary path and $W(z)$ is the adaptive filter. $x(n)$ is the reference signal,

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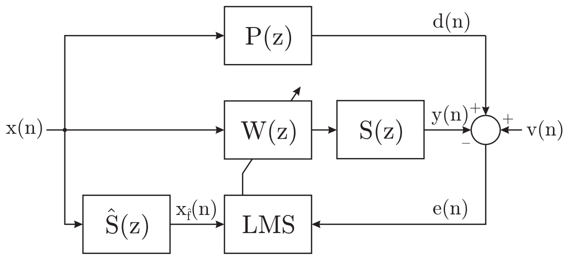

Fig. 1. Block diagram for the FXLMS algorithm

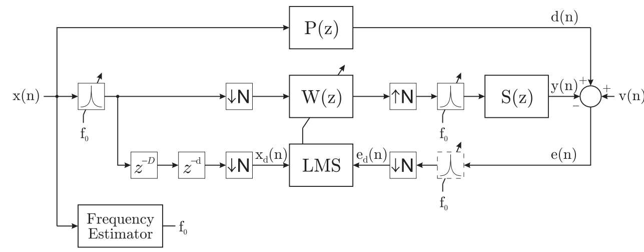

Fig. 2. Block diagram for the proposed algorithm

$x_f(n)$ is the reference signal filtered by $S(z)$, $x_{\hat{f}}(n)$ is the reference signal filtered by $\hat{S}(z)$, $d(n)$ is the primary noise at the cancellation point, $y(n)$ is the control signal, $e(n)$ is the residual noise at the cancellation point and $v(n)$ is zero mean additive noise, uncorrelated with $x(n)$.

The filtered tap vector and adaptive weight vector are defined as

$$\begin{aligned} \mathbf{x_{\hat{f}}}(n) &= \left[\, x_{\hat{f}}(n) \;\, x_{\hat{f}}(n-1) \;\cdots\; x_{\hat{f}}(n-L+1) \,\right]^T, \\ \mathbf{w}(n) &= \left[\, w_0(n) \;\, w_1(n) \;\cdots\; w_{L-1}(n) \,\right]^T, \end{aligned} \tag{1}$$

where $\cdot^T$ is the transpose operator, and the adaptive filter has length L. The adaptive filter coefficients are updated by

$$\mathbf{w}(n) = \mathbf{w}(n-1) + \mu \mathbf{x_{\hat{f}}}(n)\, e(n). \tag{2}$$

Assuming that $\mathbf{w}(n)$ converges, perfect noise cancellation is possible for K sinusoids as long as $L \geq 2K$.

Let $\mathbf{p} = \mathrm{E}\left[\mathbf{x_{\hat{f}}}(n)\, d(n)\right]$ and $\mathbf{R_{f\hat{f}}} = \mathrm{E}\left[\mathbf{x_f}(n)\, \mathbf{x}_{\mathbf{\hat{f}}}^T(n)\right]$. The mean weight behavior is given by

$$\mathrm{E}\,\mathbf{w}(n) = \mathrm{E}\,\mathbf{w}(n-1) + \mu\left(\mathbf{p} - \mathbf{R_{f\hat{f}}}\,\mathrm{E}\,\mathbf{w}(n-1)\right). \tag{3}$$

If $\hat{S}(z) = S(z)$, $\mathbf{R_{f\hat{f}}}$ reduces to the filtered signal autocorrelation matrix $\mathbf{R_f}$ and (3) has a form identical to that of standard LMS. Assuming μ is properly selected, the convergence speed is determined by the eigenvalue spread of $\mathbf{R_f}$.

We now review the $\pm 90^\circ$ stability bound, as presented in [10]. If $x(n)$ is a sinusoid with frequency ω, $S(z)$ and $\hat{S}(z)$ can be represented by the complex numbers $S_\omega = S\left(e^{j\omega}\right)$ and $\hat{S}_\omega = S\left(e^{j\omega}\right)$. We can write $\hat{S}_\omega$ in terms of S_ω, so that $\hat{S}_\omega = c_\omega S_\omega e^{j\theta_\omega}$, where c_ω and θ_ω are real constants and represent amplitude and phase estimation errors, respectively. If $P_x(\omega)$ is the power of the input sinusoid, a necessary condition for convergence is

$$0 < \mu < \frac{2\cos(\theta_\omega)}{c_\omega P_x(\omega)\, |S_\omega|^2}. \tag{4}$$

Given a secondary path delay of Δ samples ($\Delta = 0$ for the LMS case), [2] gives a popular empirical upper bound for FXLMS stability,

$$0 < \mu < \frac{1}{P_x(L+\Delta)}. \tag{5}$$

3. PROPOSED METHOD

Given (5), one would like to minimize the adaptive filter length L. For a single sinusoid, $L = 2$ is sufficient for perfect cancellation. If $\hat{S}(z) = S(z)$, $x(n) = \cos(\omega n)$ and $S\left(e^{j\omega}\right) = Ae^{j\phi}$, then

$$\mathbf{x_f}(n) = \left[\, A\cos(\omega n + \phi) \;\; A\cos(\omega(n-1)+\phi) \,\right]^T \tag{6}$$

and $\mathbf{R_f}$ is given by

$$\mathbf{R_f} = \frac{A^2}{2}\begin{bmatrix} 1 & \cos\omega \\ \cos\omega & 1 \end{bmatrix}. \tag{7}$$

The eigenvalues of $\mathbf{R_f}$ are $\frac{A^2}{2}(1 \pm |\cos\omega|)$, so that its condition number $\kappa(\mathbf{R_f})$ grows monotonically from 1 to $+\infty$ as ω varies away from $\pi/2$. In order to minimize the eigenvalue spread, one would like to sample the signal with ω as close to $\pi/2$ as possible (approximating the well-known adaptive notch filter [12]).

A block diagram for the proposed algorithm is presented in Fig. 2. In order to place the signal frequency closer to $\pi/2$, we subsample the reference and error signals by an integer factor N, which also allows significant computational savings.

Since $x(n)$ is sinusoidal, the anti-aliasing filters before decimation and after interpolation can be implemented as peak IIR filters. Aside from being computationally simple, they present zero phase shift at their center frequency. This property allows us to add or remove peak filters from the error path or reference path while keeping the FXLMS controller stable.

As the frequency of the sinusoidal noise usually changes with time, we use a frequency estimator in order to adaptively design narrow anti-aliasing filters. This has the added benefit that we can choose the optimum N on-line. An interesting frequency estimator is Quinn and Fernandes' iterative method [11], which is computationally equivalent to a 2 pole IIR filter followed by 2 multiply-accumulate operations per sample. Its output is the desired frequency conveniently expressed[1] as $\alpha = 2\cos\hat{\omega}$.

In the proposed method, all peak filters are centered on $\hat{\omega}$ and the particular application dictates their respective bandwidths. If $x(n)$ is reasonably free of noise and harmonics, the anti-aliasing filter preceding decimation may have a large bandwidth or may be removed altogether. Small bandwidths imply large group delays, implying a trade-off between tracking capability and noise rejection.

When choosing the bandwidth of the interpolation filter, one should consider the expected residual noise. Smaller bandwidths reduce noise contributions due to aliasing, but also decrease convergence speeds due to larger group delays. Given that the group delay next to a filter peak is inversely proportional to its bandwidth, series associations of peak filters can deliver better results. If ω is approximately known a priori, an optimal combination of interpolators can be designed. If the subsampling rate N is large (and the sampling rate cannot be reduced), interpolation should be performed

[1] Given a bandwidth parameter γ, 2^{nd} order peak IIR filters have the form

$$H(z) = \frac{(1-\gamma) + (\gamma-1)\,z^{-2}}{1 - (2\gamma\cos\omega)\,z^{-1} - (1-2\gamma)\,z^{-2}}$$

in multiple steps, and the same benefits of series interpolators can be obtained.

This same discussion applies to the filter applied to $e(n)$. Excellent results can be achieved without filtering $e(n)$, as long as the error does not contain sinusoids[2] at frequencies close to ω, or high frequency sinusoids whose frequencies alias to the neighborhood of ω. Sinusoids present in $e(n)$ with frequencies far from ω create a sinusoidal beat pattern in $\mathbf{w}$ which couples to the output of $W(z)$, but is filtered out by the interpolator. In general, $e(n)$ should be filtered to guarantee robustness. Filtering also prevents white noise from folding down, reducing its influence by a factor of N.

Instead of using a full secondary path model, we simply delay the reference signal. The element z^{-D} represents a coarse delay which must be estimated off-line. In scenarios with long acoustic paths, neglecting this delay line and implementing the smallest phase shift to meet the $\pm 90^\circ$ bound leaves the controller very sensitive to frequency variations in $x(n)$, forcing μ to very small values.

The element z^{-d} is a variable delay which is estimated as follows. Before the controller is initialized, the system measures the average background noise power. For values of d from 0 up to one period delay and in increments corresponding to phase changes smaller or equal to 45°, the system runs the FXLMS loop for a short time and monitors the residual noise power. If the noise power exceeds a preset threshold, divergence is assumed. Otherwise, the selected d is supposed to be within the $\pm 90^\circ$ bound for convergence. After testing all values of d, the median of all convergent values of d is selected.

Once the adaptation is running, the system estimates whether its value of d remains adequate. To do this we employ the following method: (1) put the adaptation on hold (set $\mu = 0$); (2) set $\mathbf{w}$ to $\tilde{\mathbf{w}} = (1-\delta)\,\mathbf{w}(n_0)$, where n_0 is the current sample and δ is a small positive value whose purpose will be clarified below; (3) filter the next M samples with $\tilde{\mathbf{w}}$; (4) evaluate

$$C(i) = \sum_{n=n_0}^{n_0+M-1} \tilde{\mathbf{w}}^T \mathbf{x_d}(n-i)\, e_d(n) \qquad (8)$$

for i in the neighborhood of d, where M is the test duration in samples at the decimated rate and $\mathbf{x}_d(n)$ is the decimated regressor (see Fig. 2). This test may be executed in parallel for all i; (5) update

$$d = d + \arg\max_i \; C(i) \qquad (9)$$

This procedure may be interpreted as follows. Assuming that prior to the test $\mathbf{w}$ had converged to provide cancellation, $d_d(n) \approx y_d(n) = \mathbf{w}^T \mathbf{x_d}(n-\Delta)$ and $e_d(n) \approx v_d(n)$, where the $\cdot_d$ subscript indicates decimation and Δ is the actual secondary path delay. Thus, filtering with $\tilde{\mathbf{w}}$ produces $e_d(n) \approx \delta d_d(n) + v_d(n) = \delta \mathbf{w}^T \mathbf{x_d}(n-\Delta) + v_d(n)$. Since $v(n)$ and $x(n)$ are independent, $C(i)$ is proportional to the cross-correlation between $\mathbf{w}^T\mathbf{x}(n-\Delta)$ and $\mathbf{w}^T\mathbf{x}(n-D-d-i)$, the predicted output of the control filter after traversing a secondary path with delay $D+d+i$. This cross-correlation is maximum when $D+d+i$ is closest to Δ.

The assumption that $\mathbf{w}$ had converged prior to evaluating $C(i)$ underscores the importance of having a fast cancellation loop. The measures taken along this section make this assumption practical. The choice of δ is a compromise between noise immunity, tracking capability and residual noise. Small values of δ imply longer correlation times, during which no adaptation is performed. Large values of δ increase the residual noise during the test.

[2] That is, other than the sinusoid the system is trying to supress.

Interpolator BW (rad)	NRNP (dB)	Convergence Time
$0.1 \cdot \pi$	-19.1	25 samples
$0.05 \cdot \pi$	-25.1	50 samples
$0.01 \cdot \pi$	-39.1	400 samples
$0.005 \cdot \pi$	-45.1	1000 samples
$0.1 \cdot \pi\ (2\times)$	-41.0	100 samples
$0.05 \cdot \pi\ (2\times)$	-53.2	250 samples

Table 1. Performance of the proposed method for various interpolators ($f_s = 2\,\text{kHz}$, $f_0 = 250\,\text{Hz}$, $N=2$, optimal μ)

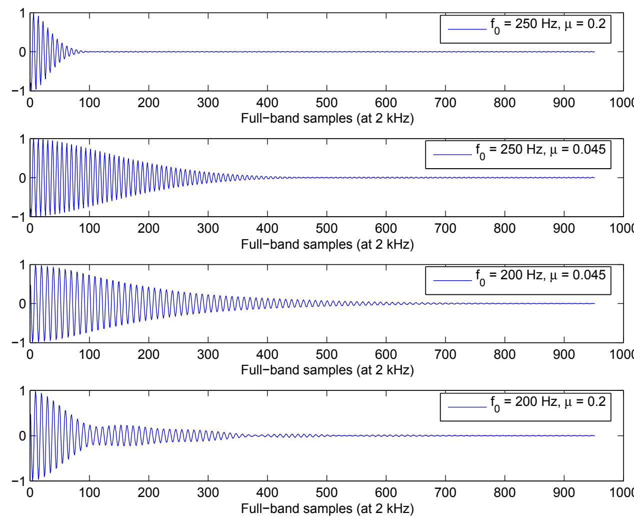

Fig. 3. Residual noise plots with $v(n) = 0$, zero delay and -40 dB NRNP. From top to bottom: 250 Hz sinusoid, interpolator composed of two peak filters, each with $BW = 0.1 \cdot \pi$ rad; 250 Hz sinusoid, single interpolator with $BW = 0.01 \cdot \pi$ rad; 200 Hz sinusoid, single interpolator; 200 Hz sinusoid, single interpolator, increased μ.

4. SIMULATIONS

The following simulations consider a sampling rate $f_s = 2\,\text{kHz}$ and subsampling rate $N = 2$. Cancellation/interpolation performance is measured with the normalized residual noise power:

$$\text{NRNP (dB)} = 10\log_{10} \frac{E\left[e^2(n)\right]}{E\left[d^2(n)\right] + E\left[v^2(n)\right]}$$

Table 1 shows a comparison between different interpolators. For these simulations, $\text{v}(n) = 0$ and the NRNP represents the noise floor due to interpolator aliasing. We consider that the adaptive filter has converged when the NRNP stabilizes.

Fig. 3 shows residual noise plots due to interpolator aliasing. For sinusoids at 250 Hz and 200 Hz we have, respectively, $\kappa(\mathbf{R_f}) = 1$ and $\kappa(\mathbf{R_f}) = 1.89$, and the convergence times differ by approximately the ratio of these condition numbers (i.e., 1.89). The bottom plot illustrates a ringing effect produced by the IIR filters for large values of μ. This initially produces fast convergence, but coefficient values overshoot their optimum and the settling time is sub-optimal, albeit by a very small amount.

Fig. 4 shows learning curves made by averaging 1000 runs with $E\left[d^2(n)\right] = E\left[v^2(n)\right] = 0.5$ and using the filters from Fig. 3.

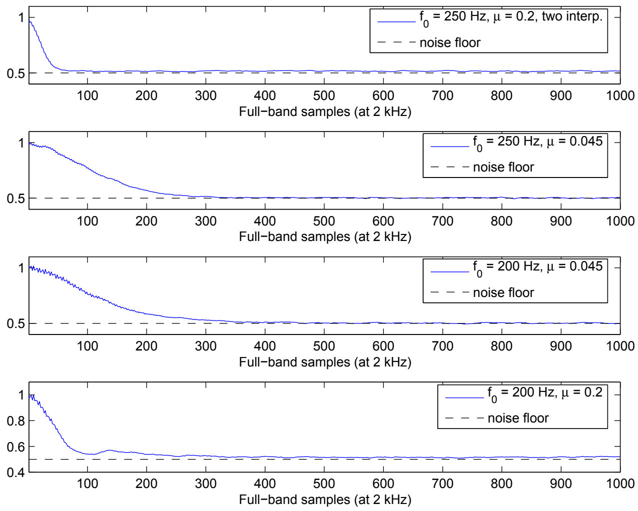

Fig. 4. $E\left[e^2(n)\right]$ curves for zero delay and $E\left[d^2(n)\right] = E\left[v^2(n)\right] = 0.5$, averaged over 1000 runs. The interpolation filters are the same as in Fig. 3.

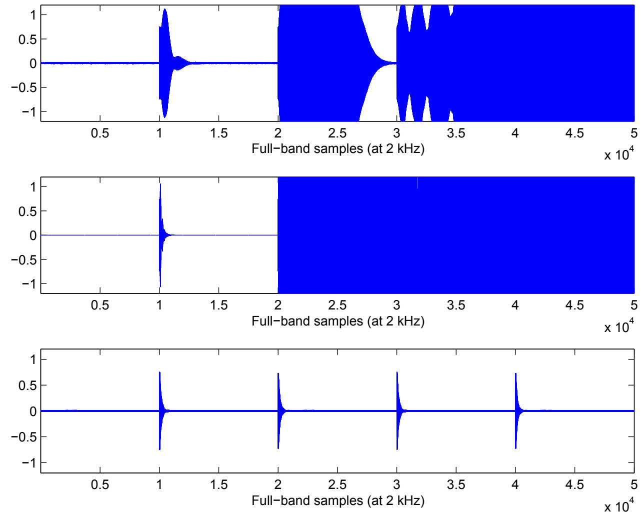

Fig. 5. Residual error under abrupt SP changes. From top to bottom: auxiliary white noise SP estimator, 2 tap LMS with no SP tracking, proposed method. All were set up to produce a -40 dB NRNP.

Note that larger adaptation steps correspond to larger steady-state errors. These plots illustrate the algorithm's robustness under very high noise levels (typically one would not apply this method with such large step sizes).

Fig. 5 compares behavior under abrupt secondary path changes for a SP estimator using auxiliary white noise [2], a 2 tap LMS filter with no SP tracking as used in [10], and the proposed method. The input was a 250 Hz sinusoid, and $S(z)$ was modeled as a 90 tap FIR filter with an impulse response featuring two exponentially decaying peaks. The auxiliary noise method was set up with an 100 tap model, and $\hat{S}(z)$ was initialized to match $S(z)$. The simulation imposes 45° phase increments on instants multiple of 10^4 samples, and despite the long interval between changes, the white noise estimator could not track them. The conventional LMS filter diverges very quickly once the 90° bound is exceeded.

5. CONCLUSION

We have presented an algorithm for tonal noise cancellation which models the secondary path as a delay. It is robust to noise and secondary-path variations, quickly tracks primary noise disturbances and has modest computational requirements. It compares very well with more traditional options for ANC, especially in applications featuring sources with small frequency fluctuations and environments with low SNRs. Its generalization to multichannel applications is straightforward, and allows sharing elements involved in decimation and interpolation. Future work includes theoretically characterizing its performance, and applying it to the open-field control of power transformer noise.

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