A Concurrent Algorithm for Blind Adaptation of DFE

M. T. M. Silva, M. D. Miranda, and R. Soares

Pós-Graduação em Engenharia Elétrica - Universidade Presbiteriana Mackenzie

Rua da Consolação, 896, Ed. Amantino Vassão - Térreo, Consolação, São Paulo, SP, Brazil

E-mails: {magnotmsilva, mdm}@mackenzie.br

Based on a concurrent deconvolution method, a new algorithm for blind adaptation of Decision Feedback Equalizers (DFEs) is proposed. It avoids degenerated solutions and achieves faster convergence and lower symbol error rate than the blind algorithms existent in the literature. Simulation results illustrate its good behavior for a digital television (DTV) channel.

Introduction: DFEs have been widely used to remove intersymbol interference from the data received through communication channels [1]. In situations where blind adaptation of DFEs is required, algorithms based on the constant modulus criterion [2] for joint updating of the feedforward and feedback filters may converge to degenerated solutions. This occurs when the signal at the equalizer output is independent of its input [3]. To avoid this problem, Szczecinski and Gei [3] proposed a new criterion based on the constant modulus cost function with constraint on the feedforward and feedback filters, which is minimized by the stochastic DFE-CMA-FB (DFE Constant Modulus Algorithm with Feedback).

In the context of blind adaptation of Linear Transversal Equalizers, based on [4], Chen [5] proposed to operate a Soft Decision-Directed (SDD) algorithm concurrently with CMA for $M$-QAM signaling. At the cost of a moderate increase in computational complexity, CMA+SDD presents an improvement in equalization performance over CMA.

Motivated by the good behavior of CMA+SDD, we propose a concurrent algorithm for blind adaptation of DFEs. It is named NDEG-SDD-CMA (Non-Degenerative SDD-CMA) and is a concurrent combination of DFE-CMA-FB and the SDD algorithm for $M$-QAM signaling.

The data model and NDEG-SDD-CMA: The signal $a(n)$, assumed independent, identically distributed, and non Gaussian, is transmitted through an unknown channel. The received signal $u(n)$ is filtered by an FIR
(Finite Impulse Response) feedforward filter of length $M_f$. The past decisions are fed back and filtered by an FIR feedback filter of length $M_b$. Then, a linear combination of the filters' outputs enters to the decision device. The equalizer input vector $u(n)$ and the decision device output vector $\hat{a}_{\tau_d}(n)$ are defined by

$$ u(n) = [u(n) \ u(n-1) \cdots u(n-M_f+1)]^T, \quad (1) $$

$$ \hat{a}_{\tau_d}(n) = [\hat{a}(n-\tau_d-1) \cdots \hat{a}(n-\tau_d-M_b)]^T, \quad (2) $$

where $\tau_d$ is a delay and $(\cdot)^T$ is the transpose of a vector.

To update the $w_f$-feedforward and $w_b$-feedback coefficient vectors, it is necessary the Lagrange multiplier $\lambda(n) = \lambda_o v(c(n))$, with $\lambda_o > 0$ and $v(c(n)) = \{1$ if $c(n) \geq 0; \ 0$ if $c(n) < 0\}$, the forgetting factor $\alpha$, the step-size $\mu$, and $R_2^a \triangleq E\{|a(n)|^4\}/E\{|a(n)|^2\}$, being $E\{\cdot\}$ the expectation operator [3, 2]. It is usual to assume $\lambda_o = 2$ and $\alpha = 0.95$ [3].

The $f$-feedforward and $b$-feedback coefficient vectors are updated from the error $\xi(n)$ with step-size $\mu_d$. To compute this error, it is necessary to divide the $M$-QAM complex plan into $M/4$ regions:

$$ A_i = \{a_{im}, m = 1, 2, 3, 4\}, \quad i = 1, 2, \cdots, M/4, \quad (3) $$

where $a_{im}$ takes the value from the $M$-QAM symbol set. Given the region $A_i$, which contains the equalizer output sample $y(n)$, the distances among $y(n)$ and the symbols $a_{im}$ must be computed as

$$ \epsilon_{im}(n) = y(n) - a_{im}, \quad m = 1, 2, 3, 4. \quad (4) $$

The parameter $\sqrt{\rho}$ should be lower than the half of minimum distance between two neighboring symbol points. The proposed algorithm is summarized as follows. We denote the complex-conjugate as $(\cdot)^*$. 
Initialize the algorithm by setting:
\[ w_f(0) = [0 \cdots 0 1 0 \cdots 0]^T, \quad E_{y_f}(0) = 0 \]
\[ f(0) = g(0) = 0, \quad w_b(0) = b(0) = 0 \]

For \( n = 1, 2, \ldots \), compute:
\[ y_f(n) = u(n)^T [w_f(n-1) + f(n-1)] \]
\[ y_b(n) = \hat{a}^T_{\tau_d}(n) [w_b(n-1) + b(n-1)] \]
\[ y(n) = y_f(n) + y_b(n) \]
\[ e(n) = (|y(n)|^2 - R_e^2)y(n) \]
\[ E_{y_f}(n) = \alpha E_{y_f}(n-1) + (1 - \alpha)|y_f(n)|^2 \]
\[ g(n) = \alpha g(n-1) + (1 - \alpha)y_f(n)u^*(n) \]
\[ c(n) = \|w_b(n-1)\|^2 - E_{y_f}(n) \]
\[ \lambda(n) = \lambda_o v(c(n)) \]
\[ w_f(n) = w_f(n-1) + \mu[\lambda(n)g(n) - e(n)u^*(n)] \]
\[ w_b(n) = [1 - \mu \lambda(n)]w_b(n-1) - \mu e(n)\hat{a}^*_{\tau_d}(n) \]

Identify the region \( A_i \) making \( 2(\log_2(\sqrt{M}) - 1) \) comparisons and compute:
\[ \xi(n) = \frac{\sum_{m=1}^{4} \exp \left[ -\frac{|\varepsilon_{im}(n)|^2}{2\rho} \right] \varepsilon_{im}(n)}{\sum_{m=1}^{4} \exp \left[ -\frac{|\varepsilon_{im}(n)|^2}{2\rho} \right]} \]
\[ f(n) = f(n-1) - \mu_d \xi(n)u^*_f(n) \]
\[ b(n) = b(n-1) - \mu_d \xi(n)\hat{a}^*_d(n) \]

**Simulation Results:** A DTV channel with large echoes fairly close to the main path is assumed [1, Fig. 8]. In the simulations, the transmitted signal and the equalizer input signal are normalized to have unit power, i.e. \( \sigma_a^2 = \sigma_u^2 = 1 \). To avoid the random phase rotation, Phase Tracking Algorithm (PTA) is considered in the implementation of DFE-CMA-FB [3]. On the other hand, it is not necessary in the concurrent algorithm, which is able to correct phase rotation.

The performance of NDEG-SDD-CMA is compared to those of DFE-CMA-FB and DFE-LMS (DFE-Least
Mean Square) without error propagation (WEP) for the DFE training with $M_f = 11$, $M_b = 17$, and 16-QAM signaling. Figure 1 shows the decision based Mean Square Error $\text{MSE} = \mathbb{E} \{|y(n) - \hat{a}(n - \tau_d)|^2\}$ of the algorithms for $\text{SNR} = 22.5$ dB. To facilitate the visualization, the MSE signals were filtered by a moving-average filter with 32 taps. The algorithms were experimentally adjusted to reach the same steady-state MSE. Observe that NDEG-SDD-CMA presents a convergence rate close to that of DFE-LMS (WEP), being much faster than DFE-CMA-FB.

Figure 2 shows the measurement of the symbol error rates (SER). As expected, the supervised DFE-LMS (WEP) algorithm has the best performance. NDEG-SDD-CMA presents a performance close to DFE-CMA-FB for signal-to-noise ratio (SNR) below 19 dB, but outperforms it for $\text{SNR} > 19$ dB, becoming close to DFE-LMS for $\text{SNR} > 22$ dB. Thus, the proposed algorithm presents a behavior close to DFE-CMA-FB for lower SNR and outperforms it for higher SNR.

Table 1 shows the computational complexity of the algorithms per iteration. The conventional CMA for blind adaptation of DFEs (DFE-CMA) is considered here for comparison. The NDEG-SDD-CMA complexity was evaluated for 4-QAM signaling. In the case of $M$-QAM, to identify the region $A_i$, it is necessary to add $2(\log_2(\sqrt{M}) - 1)$ comparisons. It is relevant to note that the computational complexity of the proposed algorithm is moderately higher than that of DFE-CMA-FB+PTA. In spite of increase of complexity, we showed by means of simulations that the resulting algorithm presents lower symbol error rate and faster convergence rate than DFE-CMA-FB.

References


Table captions:

Table 1 Computational complexity of the algorithms.

Figure captions:

Fig. 1 Decision based MSE for (i) DFE-LMS (WEP) ($\mu = 2.5 \times 10^{-3}, \tau_d = 10$), (ii) NDEG-SDD-CMA ($\mu = 10^{-3}, \mu_d = 10^{-2}, \rho = 0.06$), and (iii) DFE-CMA-FB ($\mu = 2.5 \times 10^{-4}, \rho = 0.06$); average over 50 experiments.

Fig. 2 Decimal logarithm of SER versus SNR; $\sigma_d = 10, \mu = 10^{-3}, \mu_d = 10^{-2}, \rho = 0.06$. 
<table>
<thead>
<tr>
<th>Op.</th>
<th>DFE-CMA +PTA</th>
<th>DFE-CMA-FB +PTA</th>
<th>NDEG-CMA-SDD</th>
</tr>
</thead>
<tbody>
<tr>
<td>×</td>
<td>$8M_f + 8M_b$</td>
<td>$16M_f + 12M_b$</td>
<td>$20M_f + 16M_b$</td>
</tr>
<tr>
<td></td>
<td>+19</td>
<td>+28</td>
<td>+36</td>
</tr>
<tr>
<td>+</td>
<td>$8M_f + 8M_b$</td>
<td>$14M_f + 10M_b$</td>
<td>$20M_f + 16M_b$</td>
</tr>
<tr>
<td></td>
<td>+7</td>
<td>+11</td>
<td>+25</td>
</tr>
<tr>
<td>÷</td>
<td>—</td>
<td>—</td>
<td>1</td>
</tr>
<tr>
<td>exp</td>
<td>—</td>
<td>—</td>
<td>4</td>
</tr>
</tbody>
</table>
Figure 1

![Graph showing MSE (dB) vs. iterations for three different cases: (i), (ii), and (iii).](image-url)