A new technique to construct grey-scale morphological operators using fuzzy expert system

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A new approach to grey-scale fuzzy mathematical morphology is presented. In this approach, any grey-scale window operator can be constructed using a fuzzy expert system. Many useful operators can be built using a few number of fuzzy rules.

Introduction: Mathematical morphology is one of the branches of non-linear image processing. Recently, a new theory that extends the binary to the grey-scale morphology using fuzzy set was proposed [1, 2]. These works consider a grey-scale image as a fuzzy subset of the Cartesian plane ($\mathbb{R}^2$ or $\mathbb{Z}^2$). In this Letter, an image is no more considered as a fuzzy set. Instead of that, morphological operators are represented using fuzzy expert systems. In this new approach, many useful grey-scale operators can be built using a few number of fuzzy rules. This approach also makes possible the formulation of the fuzzy morphology for arbitrary complete lattice, though in this Letter we will deal only with grey-scale morphology.

Matheron states in [3] that any translation invariant increasing binary operator can be represented as a union of erosions and a while later this result was generalised to grey-scale and to non-necessarily increasing operators. Work [2] states that, in the fuzzy morphology they developed, there are increasing operators that can not be represented as an union of fuzzy erosions. Work [4] introduces an efficient representation for operators that they called 'interval representation' (IR). This Letter generalises it, obtaining 'fuzzy interval representation' (FIR). Any grey-scale operator, including non-increasing ones, can be built using either IR or FIR, but usually FIR uses less elementary operators to represent a same operator.

Grey-scale image and window operator: Let $(E, +)$ be an Abelian group and $\mathcal{K}$ a finite chain (a chain is a totally ordered set). Then, we define a grey-scale image as a function $Q{:}E \to \mathcal{K}$, also denoted as $Q \in (\mathcal{K})^E$.

An expression composed of many window operators forms an operator used in a real image processing problem. The target of this Letter is the representation of window operators only. A window operator $\Psi{:}(\mathcal{K})^E \to (\mathcal{K})^E$ is defined via a window $\vec{W} = (W_1, W_2, ..., W_w)$, $W_i \in E$, and a characteristic function $\psi{:}(\mathcal{K})^w \to \mathcal{K}$ as follows:

$$\Psi(Q)(p) = \psi(Q(W_1 + p), Q(W_2 + p), \ldots, Q(W_w + p)),\ Q \in (\mathcal{K})^E,\ p \in E$$

Fuzzy expert system: A fuzzy subset F of an universe of discourse U is characterised by a membership function $\mu_F{:}U \to [0,1]$. The support of F

is the set of points in U where $\mu_F(y)$ is positive. A fuzzy singleton F is a fuzzy set whose support is a single point y in U and we write $F = \mu/y$. A rule is a fuzzy conditional statement of the form 'if x is F_1 then y is F_2'. A fuzzy expert system uses a collection of rules as knowledge base. Defuzzification is the process of obtaining a crisp value from fuzzy rules. For a thorough exposition, the reader is referred to [5].

Fuzzy interval representation: We can suppose $\mathcal{K} = [0\ldots k\text{-}1]$ for there is always an isomorphism from $\mathcal{K}$ into $[0\ldots k\text{-}1]$. Let Q^i, $Q^o \in (\mathcal{K})^E$ be input and output images. We will associate a real number to each element c of $\mathcal{K}$ using the 'normalisation function':

$$\boldsymbol{N}(c) = (2c+1)/(2k),\ c \in [0\ldots k\text{-}1]$$

and we will use the following 'denormalisation function':

$$\boldsymbol{N}^{-1}(C) = \operatorname{round}\left(k \max(\min(C, 1-(1/2k)), (1/2k)) - 0.5\right),\ C \in \mathbb{R}.$$

Using the functions $\boldsymbol{N}$ and $\boldsymbol{N}^{-1}$ we can suppose that the characteristic function is $\psi : [0,1]^w \to [0,1]$ instead of $\psi : (\mathcal{K})^w \to \mathcal{K}$.

We will denote a 'flat fuzzy number', a trapezoidal fuzzy subset of $\mathbb{R}$, as FFN(m_0, m_1, m_2, m_3). This quadruple of real numbers must satisfy $m_0 \le m_1 \le m_2 \le m_3$. A FFN has the following membership function:

$$\mu_{\mathrm{FFN}(m_0,m_1,m_2,m_3)}(x) = \begin{cases} 0, \text{ if } x < m_0 \\ (x-m_0)/(m_1-m_0), \text{ if } m_0 \le x < m_1 \\ 1, \text{ if } m_1 \le x \le m_2 \\ (m_3-x)/(m_3-m_2), \text{ if } m_2 < x \le m_3 \\ 0, \text{ if } m_3 < x \end{cases} \qquad (x \in \mathbb{R})$$

To represent a fuzzy subset of $\mathbb{R}^w$, let us extend the definition of FFN and obtain the multidimensional flat fuzzy number, denoted as MFFN(m_0, m_1, m_2, m_3) where $m_i \in \mathbb{R}^w$. The membership function of a MFFN is defined:

$$\mu_{\mathrm{MFFN}(m_0,m_1,m_2,m_3)}(x) = \bigwedge_{i=1}^{w} \mu_{\mathrm{FFN}(m_0[i],m_1[i],m_2[i],m_3[i])}(x[i]) \qquad (x \in \mathbb{R}^w)$$

Where $\wedge$ denotes the infimum. When a fuzzy interval primitive (FIP) $\lambda_{[m_0,m_1,m_2,m_3],V}$ is applied into a point x we get a fuzzy singleton:

$$\lambda_{[m_0,m_1,m_2,m_3],V}(x) = \mu_{\mathrm{MFFN}(m_0,m_1,m_2,m_3)}(x)/V, \quad x \in [0,1]^w, V \in [0,1]$$

Let $B = \{\lambda_1, \lambda_2, \ldots, \lambda_n\}$ be a set of fuzzy primitive intervals. Let $B(x) = \{\lambda_1(x), \ldots, \lambda_n(x)\} = \{\mu_1/V_1, \ldots, \mu_n/V_n\}$, $x \in [0,1]^w$. We will use the following defuzzification method:

$$\mathcal{d}\{(\mu_1, V_1), \ldots, (\mu_n, V_n)\} = \begin{cases} \left(\sum_{i=1}^{n} \mu_i V_i\right) \Big/ \left(\sum_{i=1}^{n} \mu_i\right), \text{ if } \sum_{i=1}^{n} \mu_i > 0 \\ \gamma^o, \text{ otherwise} \end{cases}$$

where γ^o is a given output background colour. Note that $\mathcal{d}(B)$ represents a characteristic function and we named it a fuzzy interval representation (FIR).

Representation of a window operator as a fuzzy knowledge base: To ease the construction of an operator using FIR, we defined a Pascal-like fuzzy language. We will depict below some application examples of this language. Let us declare:

```
Window = [ (0,0), (0,1), (1,0), (1,1) ];
InputBackground = 1; OutputBackground = 1;
Black = FFN(-1, 0, 0, 1); White = FFN(0, 1, 1, 2);
```

The following program was used to increase the contrast (figures 1 and 2). Its processing time was 7s in a Pentium 100 MHz. Note that the two rules below correspond to two FIP's and they represent together a characteristic function $\psi : [0,1]^w \to [0,1]$, $y = \psi(x)$.

```
if x[1] is white then y is +1.5;
if x[1] is black then y is -0.5;
```

Using 'or' operator, traditional grey-scale erosions (defined, for example, in [1]) by a flat structuring element can be constructed easily. The program below represents a 1×2 erosion and the figure 3 shows 3×3 erosion.

```
if x[1] is white and x[2] is white then y is white;
if x[1] is black or  x[2] is black then y is black;
```

Note that the second rule can not be translated into a single FIP, for it uses 'or' operator. Replacing 'and' by 'or' and 'or' by 'and' we get a flat dilation. Using weighted rules, more elaborated operators can be built. The output image of the program below is depicted in figure 4 and the processing time was 14s.

```
var d:real;
d := abs(x[1]-x[2])+abs(x[1]-x[3])+abs(x[1]-x[4]);
if d is black then y is +1.14 weight 1;
if d is white then y is -0.14 weight 5.1;
```

Conclusion: In this Letter, a new technique to construct grey-scale morphological operators using fuzzy expert systems was presented. A new representation scheme for morphological operators was introduced. Some application examples were presented.

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Fig. 1 *Input image (Q^i, 480 × 512 pixels, 1 byte per pixel)*

Fig. 2 *Output image (Q^o) with increased contrast*

Fig. 3 *Grey-scale erosion by a 3 × 3 flat structuring element*

Fig. 4 *Imitation of Adobe Photoshop 'find edges' filter*